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## NUMERICAL INVESTIGATION FOR THE NONLINEAR DYNAMIC RESPONSE OF SELF-CENTRING ROCKING FRAMES

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### ABSTRACT

Conventional seismic design standards are rooted in the notion of collapse-prevention to ensure life-safety during major seismic events. To this end, modern code-conforming buildings are designed to accept a certain level of damage during earthquakes. Nevertheless, this design philosophy does not explicitly address damage mitigation, which leads to substantial post-earthquake economic losses. Rocking post-tensioned frames are a highly sustainable seismic design solution which remains operational after an earthquake event. They capitalise on the use of: (i) unbonded post-tensioned strands to provide overturning resistance and self-centring action, and (ii) rocking (opening) joints at the column-foundation and beam-column connections. Preceding research has proposed modelling strategies to capture the highly nonlinear behaviour of rocking structures. Nevertheless, numerical modelling techniques generating frequency response functions for the study of the non-linear dynamic properties of post-tensioned rocking moment frames without any sacrificial elements or external damping, are currently limited. Thus, it is imperative to develop modelling procedures which enable an accurate representation of the fundamental non-linear dynamic behaviour of rocking frames over a range of ground-motion excitations. The physical model of a steel post-tensioned rocking frame, which has been formerly tested under static and dynamic conditions, is employed in this paper in order to validate a new advanced finite element framework. A numerical study is presented, which incorporates a one-storey model, and includes static and dynamic responses. The results are compared with experimental and discrete-element models, and generalizable modelling considerations are presented. It is shown that the proposed method encompasses a simplified modelling approach and effectively represents the complete non-linear response of rocking moment-resisting frames.

*Keywords: Rocking frames; Damage-avoidance systems; Self-centring; Post-tensioned; Nonlinear dynamics*

### 1. INTRODUCTION

Communities across our planet are repeatedly devastated by major earthquakes, which lead to economic losses associated with the widespread destruction of property, and disruption of business activities and social systems. Structures are presently designed to perform in a ductile manner, which entails that the members absorb and dissipate seismic energy by sustaining controlled deformation at designated locations, thereby, avoiding structural failure. Hence, it is irrefutable that current seismic design codes are successful in minimising the number of associated casualties. Nevertheless, damage prevention is not addressed, which impedes post-disaster recovery of the affected areas. The structural damage endured is typically deemed irreparable, and the severe financial impact from rebuilding settlements leads to a significant drop in the associated country's Gross Domestic Product (GDP)

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(Kellenberg and Mobarak, 2011). It is, therefore, evident that saving lives is not enough. We have to protect the livelihood of our communities as well.

The lasting social and economic impacts of earthquakes have propelled research into damage-avoidance earthquake resilient structures, which remain in service after a seismic event. Rocking self-centring moment frames, in particular, are damage-avoidance systems with gap-opening (rocking) connections at the base of the columns and beam-column interfaces. They utilize unbonded post-tensioning tendons to restore a structure to its initial state of zero residual displacements, in the aftermath of major seismic events. Energy dissipation is commonly provided by supplemental means rather than through the yielding (damage) of structural elements, which means that the inelastic structural deformations under lateral force can be reduced or even eliminated, in theory.

Research encompassing various aspects of the design and behaviour of different classes of rocking structures has spanned over the last two decades (Ricles et al. 2001), (Roke et al. 2009) (Eatherton et al. 2010) (Newcombe, 2011) (Wiebe et al. 2013) (Clayton et al. 2014), (Dejong and Dimitrakopoulos, 2014), (Vassiliou et al. 2016). Nonetheless, there aren't many studies examining the fundamental non-linear behaviour using frequency response functions of post-tensioned steel rocking frames, independent of any yielding components and damping devices. Similarly, there have been pertinent developments proposing modelling techniques to capture the highly non-linear behaviour of rocking frames, utilising finite and discrete element frameworks. However, the majority of this work has considered the resilience of individual joint assemblies, following a concentrated plasticity approach, combined with a hysteresis rule idealisation (flag-shaped). Moreover, most investigations have focussed on braced and shear wall structural configurations. Research conducted on moment frame structural assemblies to study overall interactions is scarce. To this end, it is critical to develop standardised procedures with generalizable modelling considerations. Finite element modelling procedures are formulated herein for a single-storey post-tensioned rocking moment frame. These methods are further validated against the experimental, analytical, and discrete-element responses of a selected single-storey structure. The emergence of an effective modelling methodology can simplify the prediction of structural behaviour, and ultimately facilitate the practical application of these structures.

## **2. RELEVANT LITERATURE**

Housner (1956, 1963) and Muto et al. (1960) marked the advent of rocking structures by conveying the potential economies of utilizing the rocking mechanism for damage-resistant seismic design. They observed that slender inverted pendulum type structures survived major earthquake events with only minor damages, while more stable appearing structures experienced overturning. Thereafter, Clough and Huckelbridge (1977) carried out feasibility studies of rocking building structures, comprising experimental and numerical investigations of multi-storey reduced scale steel frames with base-uplifting columns. The finite element program, Drain 2D was utilized for numerical modelling, with the uplift response represented using bilinear elastic support elements with zero tensile stiffness and force capacity in the upward direction. It was concluded that the rocking motion did not provide adequate damping to the structural system, resulting in significant storey displacements. Stanton (1987) further introduced the possibility of utilizing unbonded post-tensioned strands in pre-cast concrete connections for seismic resilience and stability, thereby providing evidence of the compelling benefits of structures featuring controlled rocking connections.

The promising results from former studies prompted the substantial PRESSS (Precast Seismic Structural Systems) program, which investigated an array of alternative connections for precast concrete frame and wall structures. The base purpose of the program was to inspire the usage of precast concrete systems in high-risk seismic areas (Priestley, 1991). The program culminated with the test of a 60%-scale, five-storey two-bay building which incorporated: a hybrid coupled wall to provide lateral force resistance in one direction, moment-resisting frames with and without unbonded tendons in the other direction, and components exhibiting rocking, at beam-column or wall-foundation

interfaces. The structure withstood events up to 50% greater than the design level event with minimal cosmetic damage (Priestley et al. 1999) (Sritharan et al. 2000) (Chancellor et al. 2014). Therefore, it was demonstrated that the post-tensioned moment-resisting frame was a robust structural system which minimized post-earthquake damage, and had the ability to accommodate large deformations with minimal residual drifts.

Following the success in concrete, the reliability of integrating the PRESSS concept in steel structures was investigated. Subassemblies of beam-column moment-resisting connections for post-tensioned rocking frames with wide-flange sections were experimentally tested by Garlock et al. (2002, 2005). Moreover, numerical investigations were carried out to examine the seismic behaviour of a 6-storey, 4-bay post-tensioned steel moment-resisting frame with sacrificial bolted top and seat angles in the connection to provide energy-dissipation. With the exception of minor design adjustments, it was concluded that post-tensioned tendons in steel rocking moment connections, provided sufficient strength, stiffness, and drift capacity under dynamic loading. The non-linear dynamic behaviour of single-storey post-tensioned moment frames with rocking connections at beam-column and column-foundation interfaces were explored in (Oddbjornsson, 2011) (Oddbjornsson et al. 2012) (Alexander et al. 2011), using physical, analytical, and numerical modelling, exclusive of supplemental energy-dissipation components. The quarter-scale physical model represented a single bay and storey from the PRESSS building (Priestley et al. 1999). The Discrete Element (DE) software UDEC (Itasca, 2014), was used to capture the fundamental nonlinear mechanics of the physical model (Oddbjornsson, 2011). Empirical models were further developed using energy considerations to represent the equivalent single-degree-of-freedom (SDOF) non-linear dynamic response (Alexander et al. 2011). It was, henceforth, stated that the DE model adequately replicated the qualitative behaviour of the physical and analytical models. Nevertheless, the discrete-element models were observed to be highly sensitive to input parameters concerning material properties and damping models. Therefore, the input sensitivities associated with discrete-element modelling may limit the reliability of the framework in context of modelling non-linear rocking frames.

From the foregoing, it is evident that numerical modelling approaches in both discrete and finite element frameworks have been presented to represent the rocking motion of frames. In essence, the numerical modelling of rocking post-tensioned moment-resisting frames presents three major challenges: i) Accurate representation of the non-linearity associated with the joint opening mechanism, ii) Modelling the anchorage points for the post-tensioning strands, iii) Material models for members and opening interfaces. The multi-spring contact element approach, conceptually equivalent of a Winkler soil foundation model, is the most commonly implemented strategy in finite element simulations for the in-plane rocking motion. The geometry of this element comprises of an evenly distributed series of zero-length elements, confined between rigid links at the end surface of the adjoining structural members. The gap material defined typically has no resistance in tension and an elastic response in compression. This modelling approach results in the contact zone being infinitely stiff, and the opening joints rotating around the pivot point at the member edges. Therefore, the motion simulated, as a result of this modelling methodology, conforms to the rocking principles introduced by Housner (1956).

Spieth et al. (2004) presented an alternative approach for the numerical modelling of rocking connections, using the finite element program, RUAUMOKO (Carr, 2007). The purpose of the study was to develop a strategy that allowed the simulation of compression zones which develop in the end-segments of reinforced concrete rocking beam-column connections. The location of springs and the distribution of stiffness along the contact area was calculated based on Gauss and Lobatto integration schemes. The numerical model was compared with experimental results of beam-column connection sub-assemblies and a multi-storey reinforced concrete frame structure with draped tendons. The method was demonstrated to successfully represent the non-linear joint behaviour of the connections, and capture the beam elongation and post-tensioned tendon lengthening effects. In addition, the strategy permits the modelling of local damage in the contact zone. It was also concluded that the global results obtained using the two integration formulations were similar, and the schemes can be selected and applied interchangeably. Hence, it is evident that the favourable characteristics of rocking



structures has captured the interest of a significant number of researchers. The descriptions and technologies presented herein indicate that there is a need to propose a finite element modelling strategy capable of predicting the complete non-linear dynamic response of rocking steel moment frames. The following paper presents finite element investigations using the program OpenSees (McKenna, 1997), of a single-storey rocking post-tensioned moment frame. The non-linear response of the frame is studied exclusive of external energy-dissipation, numerical modelling procedures are developed, and comparative analysis is carried out with the selected experimental and numerical benchmark studies.

### 3. BENCHMARK STRUCTURE

#### 3.1 Rationale

The following numerical investigation required a well-documented physical rocking moment-resisting frame benchmark study. This model would be used to validate finite element models and provide a theoretical background for the forthcoming studies. For this purpose, the model had to be extensively explored for both static analysis and dynamic excitation. Structural simplicity was an additional objective included in the selection criteria. The simplicity of a structure in this case was evidenced by structural regularity and the absence of complex supplementary energy-dissipation elements, devices or mechanisms. This would reduce the uncertainties that arise in the modelling and analysis stages, enabling a more fundamental understanding of the dynamic phenomena. The purpose of these objectives was to allow the study of a prototype structure which exhibits non-linear behaviour that can be predicted with a good degree of reliability, thus allowing an accurate study of the relevant non-linear effects in the numerical models. In consideration of these attributes, the model described in (Oddbjornsson, 2009) (Oddbjornsson et al. 2012) (Alexander et al. 2011), was chosen for this research. The aforementioned series of investigations were among the first to study in detail, the true rocking response of a complete moment frame structural assembly. Therefore, this study provides feasible baseline results to validate subsequent numerical models.

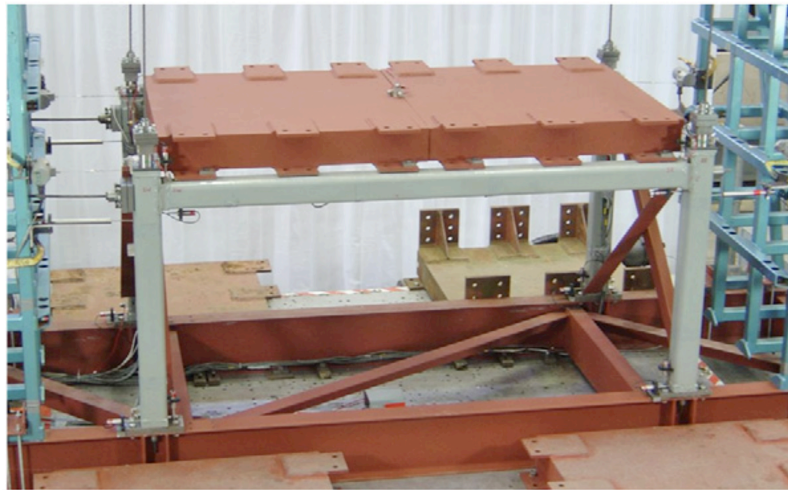


Figure 1. The physical structure (Alexander et al. 2011).

#### 3.2 Model Description

The structural model employed is a one-storey quarter-scale single-bay planar steel frame, comprising of moment-resisting rocking connections at the beam-column and column-foundation interfaces (Figure 1). The height of the frame above ground is 0.9 m and bay width is 2.1 m. These values correspond to an aspect ratio of 0.4. The detailed design of the original model building was carried out in accordance with design guidelines presented as a part of the PRESSS research programme

described in the background study (Sritharan et al. 2000). To this end, the physical model studied in the preceding research was based on a typical light-weight portal frame building prototype, representing a single bay and storey from the PRESS structure. The beam and column elements of the model frame were both made from hollow steel square sections, with dimensions of 100 x 100 x 10 mm<sup>3</sup>. A tendon pre-tension and cross section of 115 kN and 93 mm<sup>2</sup> for beam tendons, and 64 kN and 52 mm<sup>2</sup> for column tendons was determined. The applied mass on the frame was two tonnes, representing the seismic inertia mass. The specimen was tested at the University of Bristol Laboratory for Advanced Dynamics Engineering (BLADE).

## 4. NUMERICAL MODELLING STRATEGIES

### 4.1 Modelling Specifications

A planar model with two-degrees-of-freedom per node was defined. The coordinates of the frames were specified in accordance with the geometric details presented in Figures 1, 2, 3, and 4. Both the Lobatto and Gauss integration schemes introduced by Spieth et al. (2004), were applied and compared. Varying number of springs were modelled ranging from 2 to 10 springs in order to assess the effects of changes in stiffness distribution at the base. All degrees of freedoms were constrained at the foundation level to simulate a fixed base beneath the rocking interface. The end node at the base opening joint was horizontally restrained to simulate a roller support in the X-direction. Vertical displacement restraints were added for the zero-length elements between the beam and column to facilitate shear transfer. This was achieved by using the equal degree of freedom constraint (master-slave nodes) for adjacent nodes between the beam-end rigid links simulating the gap opening. Nodal masses were lumped at the top of each column element and defined in the horizontal and vertical degrees of freedom. Corotational geometric transformation was applied for columns and beams to account for geometric nonlinearity, and linear transformation was used for the rigid links.

The columns and tendons were modelled as continuous linear elements along the frame heights. The post-tensioning tendons were modelled using Corotational truss elements with an initial stress uniaxial material (Steel02 material in OpenSees), and were anchored at points rigidly offset half the column and beam depth outside the frame centreline (Figures 3 and 4). An alternative model was developed for post-tensioning anchored at the frame centreline in order to study the effects of the offsetting (Figure 2). The frame and post-tensioning elements were modelled with their respective material elastic moduli and yield strengths. Elastic beam-column elements were used for modelling the beams, columns, and rigid links. Elastic Perfectly Plastic material was defined for the gap element, with no stiffness in tension. Alternative models of gap elements included the Elastic No-Tension material, and hysteretic material. It was observed that material selection for the gap elements did not have a major impact on the dynamic results. The stiffness properties of each spring element were calculated using Equations 1 and 2 below:

$$K_{spring,i} = \frac{EA}{2L_i} * w_i \quad (1)$$

$$F_{S_{spring,i}} = \frac{F_y A}{2} * w_i \quad (2)$$

Where, E, A and F<sub>y</sub> is the Modulus of Elasticity, Cross-Sectional Area, and Yield Strength of the connecting member; w<sub>i</sub> is the weight of each spring assigned in accordance with Lobatto integration scheme listed in Table 1; L<sub>i</sub> is the influence length. Influence lengths of 250 mm and 350 mm were used for beams and columns, respectively. These influence lengths were selected and calibrated based on the results of the experimental study. The recommended baseline for these parameters is approximately one-third of the storey height for the columns and one-sixth of the beam length for the beams.

Table 1. Spring Position and Weighting for a 10-spring gap element (Spieth et al. 2004).

No. of Springs	Gauss Integration		Lobatto Integration	
	Abscissas	Weights	Abscissas	Weights
10 Springs	$\pm 0.149$	0.296	$\pm 0.165$	0.328
	$\pm 0.433$	0.269	$\pm 0.478$	0.292
	$\pm 0.679$	0.219	$\pm 0.739$	0.225
	$\pm 0.865$	0.150	$\pm 0.920$	0.133
	$\pm 0.974$	0.067	$\pm 1.000$	0.022

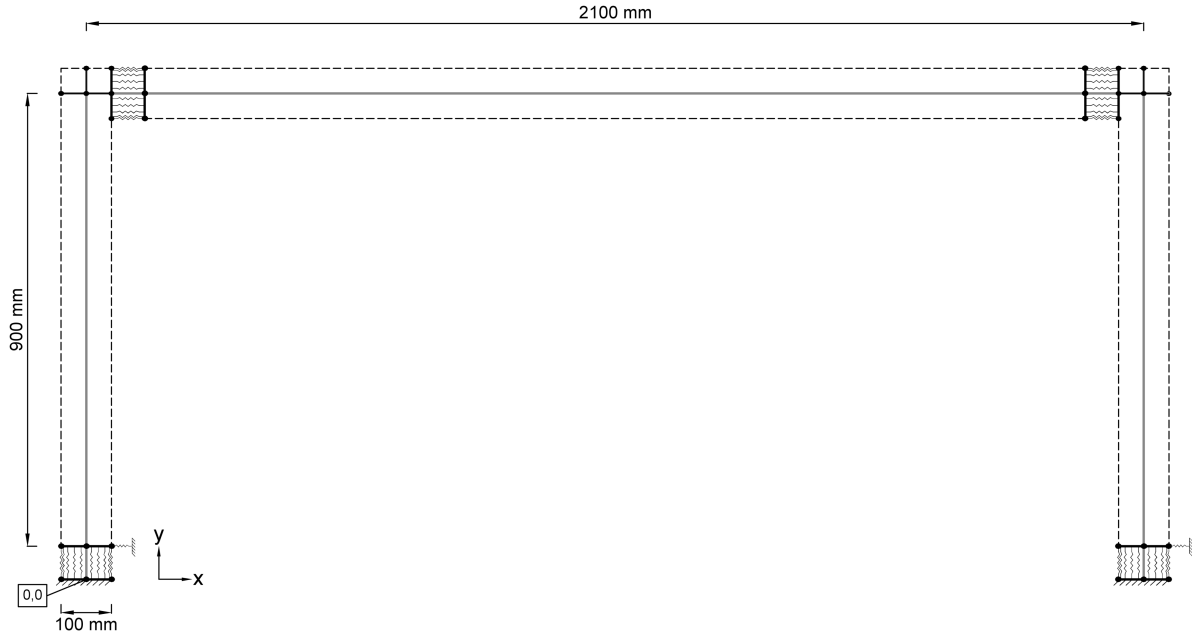


Figure 2. One storey, single bay model schematic with members and connection details.

#### 4.2 Analysis Parameters

The Static and Dynamic analyses include the definition and application of gravity loads, static lateral loads, and dynamic loads. Beam gravity loads were applied as nodal loads on columns in order to simplify the model and decrease processing time for convergence. Distributed beam loads were additionally investigated and observed to not have an impact on the static and non-linear dynamic results of the system. A displacement control strategy was used to perform the pushover analysis where, a target displacement and displacement increment were defined for each model. Stiffness proportional damping of 5% was specified, using the fundamental frequency of the structure, calculated from the first eigenvalue.

Resonance response curves for structural engineering applications are typically generated using a sine-sweep or, more accurately, a sine-dwell input ground motion. Sine-dwell analysis is the discrete version of a sine-sweep analysis, where the frequency is not varied continuously but is incremented by discrete amounts, giving the structure time to reach a steady-state response. MATLAB (MathWorks, 2004) was used to generate input harmonic ground motions for the numerical models. An algorithm was programmed with an input interface which comprised of variables including frequency range, time increment and sampling rate. Based on the input variables, the code generated an acceleration-history, which was imported into OpenSees.

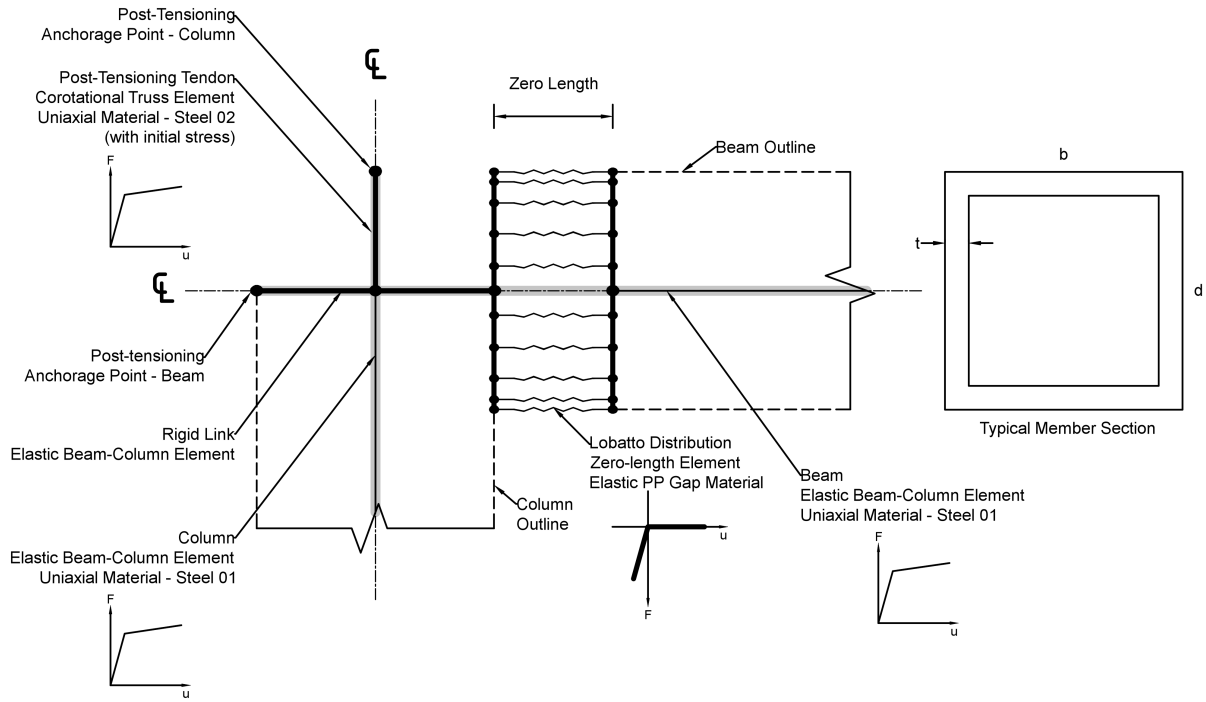


Figure 3. Beam-column connection detail with Post-Tensioning (PT) rigidly offset from centreline.

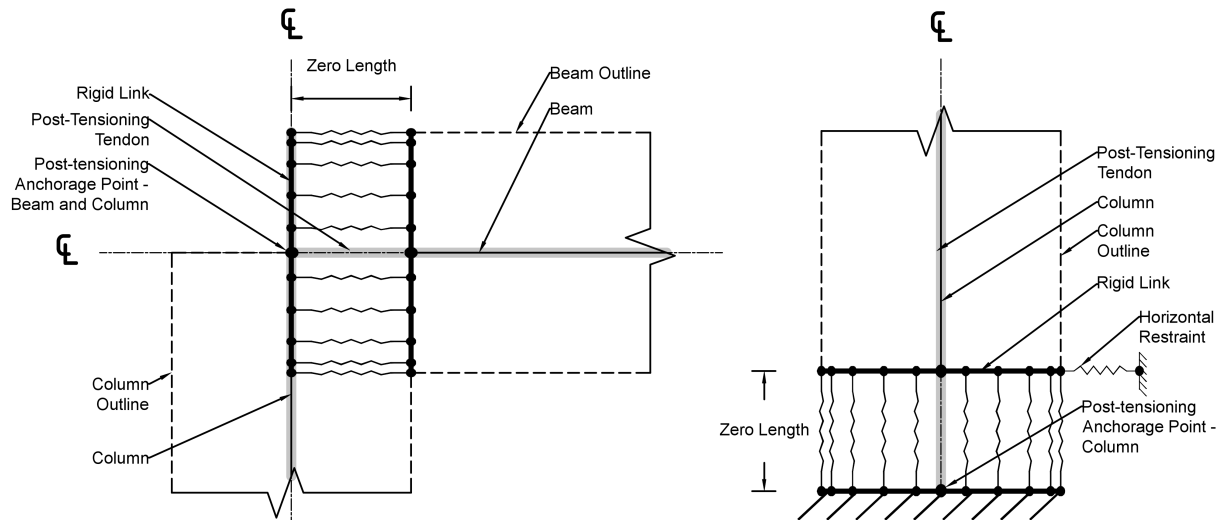


Figure 4. Beam-column connection without PT anchorage offset (left); Column-foundation connection (right).

## 5. RESULTS AND DISCUSSION

### 5.1 Static Analysis

The results of the static analysis for the single-storey model are illustrated in Figure 5. Post-processing of data and generation of the response plots was carried out using MATLAB (MathWorks, 2004). The response displacement for the top storey was plotted against the base shear normalised using the weight of the frame. Sensitivity analyses were carried out for structural parameters such as the number of gap elements, influence length, post-tensioning anchorage offsets, and pretension force. Through comparisons with the experimental study, it was demonstrated that the numerical model qualitatively represented a realistic form of the push-over curve for a rocking post-tensioned moment frame

structure. The experimental and the initial push-over analysis results were used to calibrate the elastic stiffness of the frames by altering the influence length values shown in Equation 1. It follows from Equation 1, that a model with a lower value for the influence length leads to a stiffer rocking zone and a higher ultimate load for the push-over analysis. It was further observed that altering the initial pretension force of the columns did not have a significant impact on the static response, whereas increasing the initial pretension force of the beam by a factor of 1.4 resulted in a 25% higher ultimate load. It was also noted that the anchorage points offset did not have a significant influence on the pushover response of the structures. For each model, varying number of spring elements ranging from 2 to 10 were defined to determine the optimal number of zero-length elements. Figure 5 illustrates that as the member stiffnesses along the rocking surface are more distributed, the pushover curves become smoother, ultimate load decreases, and the curves begin to overlap. This implies that an asymptote is reached as the number of springs increases, where distributing the member stiffness further along the surface does not yield any change in the results. A recommendation is made that that a suitable baseline value for the number of gap elements is to divide the width of the rocking surface by a factor of 10. The selected number can be further calibrated based on the initial static results obtained.

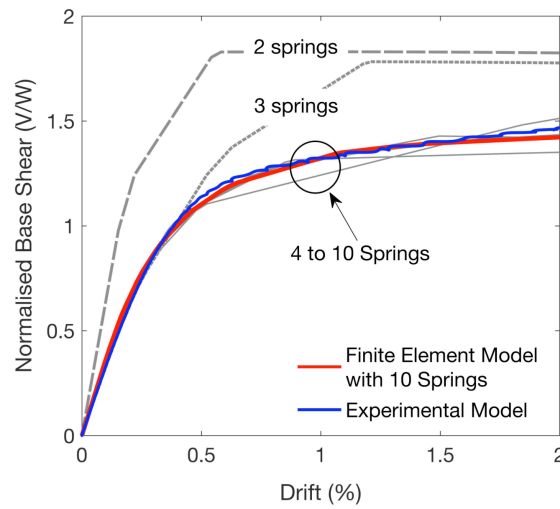


Figure 5. Pushover Response of Physical and Numerical Models for 2 to 10 springs in the contact zone.

## 5.2 Dynamic analyses

A series of dynamic analyses were performed for varying forcing amplitudes. The dynamic data was post-processed using MATLAB (MathWorks, 2004), where the displacement response-history obtained from the discrete sine-sweep analyses, was converted to the frequency domain. The frequency response functions (forcing frequency versus drift) were developed in this manner and plotted in conjunction with the physical and discrete-element results from Alexander et al. (2011). The lower-amplitude branch for the preceding physical and discrete-element models, was obtained using an increasing frequency sine-sweep and the upper branch using a decreasing frequency sine-sweep analysis. In contrast, the initial Finite Element model resulted in identical responses for both the increasing and decreasing sine-sweep analyses. The distinct upper and lower branch resonance responses were obtained by introducing a high amplitude excitation for a finite duration, prior to the application of the desired frequency sweep loading. The purpose of incorporating the impulse loading was to alter the initial conditions of the non-linear frame (e.g. velocity) at the time the sine-sweep motion is applied, resulting in the upper branch response.

Figures 6 illustrates the high amplitude frequency response functions for the single-storey physical and numerical (finite and discrete element) models. The forcing frequencies were normalised against the natural frequency of the finite element model. It can be observed that the Finite Element model captures the features of the physical model very well. It is also noted that the responses of the discrete-

element model are divergent from the physical model, particularly at low amplitude forcing. Moreover, the resonance curves for the physical model do not match the numerical responses, at lower amplitudes. A possible reason for this might be the difficulty experienced during experimentation to control the test specimen around resonance. Sensitivity analyses of the numerical models also revealed that the initial pretension force does not have an effect on the dynamic characteristics of the models. This was also observed in the study conducted by Alexander et al. (2011).

The sine-sweep curves were further plotted for a range of forcing amplitudes. A backbone curve was constructed by connecting the highest response coordinates in the displacement-based resonance response plots. For nonlinear systems, a feature of backbone curves is that the peaks of the low amplitude responses, approach the fundamental frequency of the system (Alexander et al. 2011). Thereafter, a series of dynamic analyses were performed on the numerical model with sinusoidal loading of low, medium and high amplitudes at the resonant frequency. The results were compared with the numerical pushover response obtained previously (Figure 7). The resulting plots demonstrate that the structural system does not inherently possess any energy-dissipation (damping) capability, as the area of the dynamic sinusoidal response follows the pushover backbone curve.

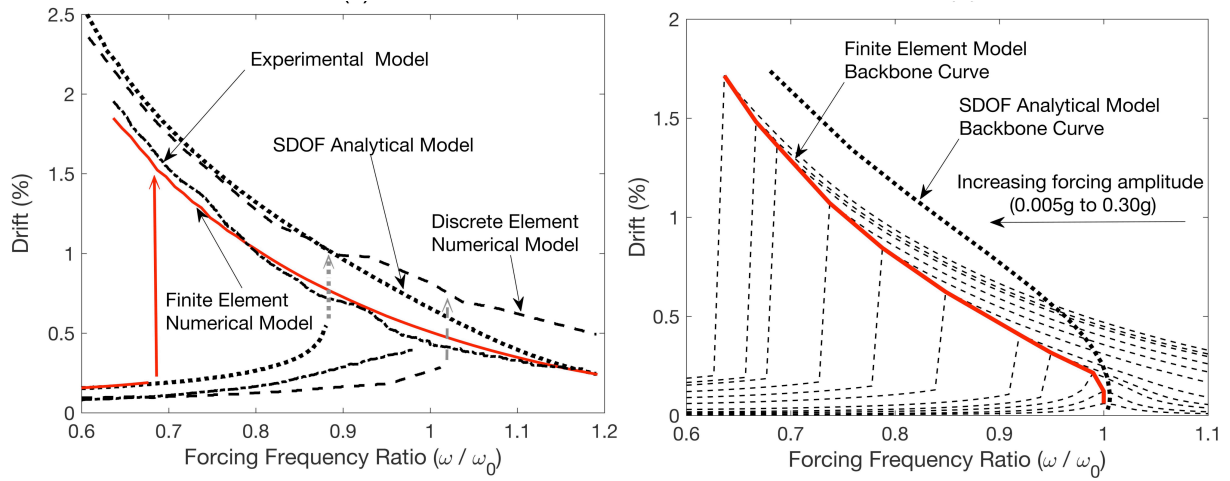


Figure 6. Frequency Response Functions for a high forcing amplitude of 0.26g (left); Backbone Curves (right).

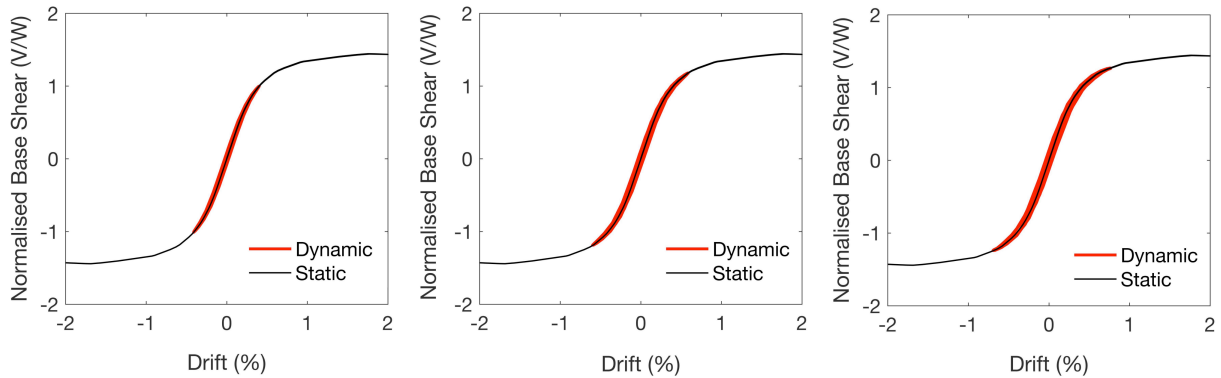


Figure 7. Hysteresis curves for cyclic loading amplitudes of 0.10g (left), 0.20g (centre), and 0.30g (right).

## 6. CONCLUSIONS

Numerical modelling of post-tensioned rocking frames was performed using the finite element framework, OpenSees. A single-storey benchmark rocking moment-resisting frame structure was selected from literature. Two-dimensional numerical models of the post-tensioned moment frame were formulated in OpenSees to simulate the results of the experimental tests. The numerical modelling

techniques implemented, produced static and dynamic results which were qualitatively and quantitatively similar to the preceding experimental and analytical studies. Moreover, detailed strategies to obtain the frequency response functions for rocking moment frames using finite element modelling were discussed. The resulting numerical model was not highly sensitive to the input parameters which illustrates the reliability of the proposed modelling methods. These strategies will further be applied to multi-storey rocking frames and studied extensively. A series of ground-motion analyses will also be carried out in subsequent studies to examine the seismic response of post-tensioned rocking frames.

## 7. ACKNOWLEDGMENTS

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